# Satisfying Simultaneous Resolution and Noise Criteria in Digital Images

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## Abstract

The resolution and noise properties of digital images are crucial components of overall image quality. Individual physical metrics are plentiful for both of these attributes, as are psychophysical descriptors for the corresponding visual effects, but without necessarily providing any cohesive insight into the technical parameters driving overall quality. The author has recently described Fourier-based metrics especially suitable for the evaluation of digital images, and has developed absolute scales that allow direct comparisons between different digital technologies, and which also readily tie-in with older analog quality scales. Here this analysis is extended to the practical problem of simultaneously meeting digital resolution and noise criteria, within technical limitations imposed by parameters such as print dpi and number of available gray-levels.

### Introduction

The quality attributes of images formed on a grid, raster, or similarly discrete geometrical framework have been studied for more than half a century.<sup>1-6</sup> These studies have spanned numerous technologies and applications, (eg television, printing, electrophotographic halftones, satellite reconnaissance and medical diagnostic imaging), and the underlying analysis has included appropriate contributions from Fourier analysis, information and signal-detection theories and visual science.

During recent years the author has attempted to distill the results form these earlier studies into a systematic set of image quality descriptors appropriate for example to inkjet printing<sup>7,8</sup>. In this way an absolute scale has been described for digital noise and a similar scale developed for digital sharpness. The digital noise scale (DNS) has both a direct visual-science Fourier-basis yet lends itself to practical physical measurement, and in addition has the advantage that it is directly related to long-established granularity metrics in analog photography, and can also be simply translated into key digital printing parameters such as dpi and number of gray-levels. In similar fashion an absolute scale has been described for digital sharpness (DSS) that can be directly related to the sharpness associated with other imaging technologies (such as analog photography) and translated into pixel-size/enlargement

terms as for example appropriate for ink-jet printing of digital photographs.

The aim here is to combine the results of these studies into an overall model for the image quality of ink-jet printing, thus allowing simultaneous criteria to be defined for both resolution and noise. In this way the implications on overall quality imposed by the practical gray-level/dpi range can be clearly demonstrated, and used both to specify quality levels and set quality targets. This is especially relevant in light of increasing interest in the systems combination of digital acquisition and printing technologies. But first it is appropriate to summarize the assumptions behind the digital noise and sharpness scales.

## A Scale for Digital Noise

As first developed for photographic granularity and later extended to electrophotography, and subsequently translated into convenient digital form, the digital noise may be expressed on the *DNS* as

$$DN = \sqrt{\left\{ \iint WS_{R}(u,v) VTF^{2}(u,v) \, du \, dv \right\}}$$
(1)

where  $WS_R(u,v)$  represents the Wiener Spectrum of the noise fluctuations measured in units of reflectance, and VTF(u,v) denotes the transfer function associated with human vision. We adopt here a visual transfer function, assumed standard for normal print viewing conditions<sup>2</sup>, as shown in figure 1.



Figure 1. The assumed visual transfer function for standard print viewing condition

In many practical cases it is possible to simplify this expression to

$$DN = \sqrt{WS_{R}(0,0)} \tag{2}$$

The author has also indicated that existing empirical descriptors for photographic grain fall on the DNS as below, implying a gamut of physical values in the range I to I0 for practical photography.

DNS	<u>Photo-Grain</u>
10	off-scale
8	very coarse
6	coarse
5	moderately coarse
4	medium grain
3	fine grain
2	very fine
1	extremely fine
<1	microfine

A simple model for the image noise associated with ink-jet printing may be approximated on the digital noise scale in terms of *dpi* according to

$$DN(max) = \frac{12,700}{(m \, dpi)}$$
 (3)

where m denotes the number of available gray-levels expressed in reflectance-space.

Equation (3) demonstrates the equivalent roles played by dpi and availability of gray-levels in reducing digital noise. This equivalence is illustrated below in Figure 2, where the noise due to a coarse binary image (top left) is reduced by successive doublings of dpi (horizontal) and gray-levels (vertical), and combinations of the two.



Figure 2. Illustration of the influence of dpi and gray-levels on digital noise (see text).

#### A Scale for Digital Sharpness

In constructing this scale we assume the same visual transfer function, but must now consider the introduction a spatial-frequency spectrum that will act as a global surrogate for those aspects of the input (scene) which convey the impression of sharpness. For this we assume a flat (white) scene-spectrum and the resulting product of this spectrum and the visual transfer function is shown in Figure 3. Note that due to an assumption of circular symmetry we have reduced the spatial frequency from two-dimensions (u,v) to one (w), by effectively changing to polar coordinates and hence introducing the radial multiplier (w) in the product. The same result is obtained by assuming a one-dimensional (line) scene-spectrum and assuming a linearly-increasing scene-spectrum.



Figure 3. The assumed visual spatial-detail detection-function

For the present purposes the transfer function associated with the digital printing process is considered to be due entirely to the pixel grid structure and can therefore be represented by a *sinc* function based on the pixel dimensions in the standard. Figure 4 shows this function for pixel sizes of 4, 8, 16, 32, 64, 128 and 256 microns.



Figure 4. The pixel-grid transfer-function for various pixel sizes.

The transfer function for the pixel array is now combined with that of figure 3 to yield an overall spectrum for the spatial-detail detection function, for this same range of print pixel-dimensions. Since the smaller pixel-sizes have spatial frequency band-passes far beyond that of the visual system, the curves shown in figure 5 crowd together for these small pixel sizes, the limiting curve of course being simply that of figure 3.



Figure 5. The overall spatial-detail detection-function

We now hypothesize that the spatial-frequency integral of the above curves as a metric of perceived print sharpness, or the digital sharpness scale (DSS). In other words we define digital sharpness (DS) according to the one-dimensional (line) integral

$$DS = \int pixTF(w) VTF(w) w \, dw \tag{4}$$

In the absence of a closed-form solution, numerical integration yields the digital sharpness curve shown in figure 6 as a function of print pixel size.



Figure 6. Digital sharpness as a function of print pixel size

For convenience the scale has been normalized to 10 for an arbitrarily small pixel (ie, the integral of the function shown in figure 2), yielding a convenient 0 to 10 scale for the complete gamut of sharpness values. It should be stressed here that the pixel size refers to that effective in the viewed print, and in digital photography this may be greater than the basic print-resolution dimension - and is always almost greater than the pixel dimension associated with image acquisition in the camera.

Figure 7 shows the result of figure 6 expressed in the more familiar print terms of pixel resolution (*ppi*). From this we note that according to this new scale there is an almost linear increase in sharpness up to around 150*ppi*. Thereafter further increases in *ppi* bring diminishing sharpness benefits, while beyond 600 ppi print sharpness approaches its upper limit in asymptotic manner.

For the sake of context the range of this scale can be illustrate by estimation of the sharpness values associated with consumer analog photography. For this the equivalent pixel-size in the negative is assumed to fall within the range of 5 to 10 microns - practical values estimated from spread function diameters of typical modern negative materials. Secondly, the practical format/enlargement range of interest is assumed to fall between the extremes of APS format enlarged to 8" inch prints and 35mm format to 3.5"prints. Combining all these assumptions leads to an estimation for the practical range of spread-functions as falling between 20 and 120 microns in the analog print, with corresponding sharpness values varying between 8 and 9.95 according to the digital sharpness scale.



Figure 7. Digital sharpness as a function of print ppi.



Figure 8. Relationship between number of sensor x-pixels and print x-dimension in order to conform to the range of sharpness values typical for analog photography.

A useful way of scaling these analog values alongside key parameters in digital photography is as shown in figure 8. Here the number of acquisition pixels on a side and the physical size of the print on this same side have been used as surrogates for print pixel size, and plotted according to sharpness criteria in the analog photography sharpness range. Thus according to any desired sharpness criterion it is possible to understand the maximum print size that will meet this criterion for a specific acquisition array size. For example, we see that a sharpness value of 9.5 for an 8" print implies almost 4000 pixels on a side.

# Simultaneous Resolution and Noise Criteria

We are now able to consider the *mutual* properties of digital noise and digital sharpness in the print *ppi* and *graylevel* domain, since we have reduced both these imagequality attributes to simple models within this same domain. Figure 9 shows *gray-level/ppi* performance curves on the digital noise scale, where each *ppi* is now associated with a specific value of digital sharpness, as shown. Figure 9 thus acts as a means of understanding the implications when simultaneously setting print image-quality targets for noise and sharpness. An example of this is given in Figure 10. For this example it has been assumed that achieving a noise level of 1.5 or less is desired, along with a sharpness level of 8.5 or higher.



Figure 9. Digital noise (y-axis) as a function of available graylevels (x-axis) for a range of print ppi values as shown. Also shown are the corresponding digital sharpness values.



Figure 10. Digital noise (y-axis) as a function of available graylevels (x-axis) for print ppi values as shown. The horizontal line denotes DN=1.5 while the dashed curve is for DS = 8.5.

From Figure 10 we see that the imposition of these joint noise and sharpness criteria results essentially in four regions as bounded by the dashed lines. The top left region constitutes a region of excessive noise, although meeting the sharpness criterion, while the top right region implies both excessive noise and lack of requisite sharpness: the bottom right region represents lack of sharpness, although the noise is satisfactorily low. Only within the bottom left region can both the sharpness and noise criteria be met simultaneously, thus defining the appropriate combinations of ppi and gray levels which may be used to stay within specification. Although only included for the purposes of illustration, it should be pointed out that the specific numerical values chosen for noise and sharpness are perhaps representative of a quite sophisticated level of conventional photographic quality.

# **Summary and Conclusions**

Image quality metrics appropriate for digital prints have been reviewed, and simple models have enabled these metrics to be interpreted in terms of the print parameters representing gray-level and *ppi* characteristics. From this it has been concluded that it is possible to simultaneously specify both print noise and sharpness, and to translate these joint requirements into practical *gray level* and *ppi* domains. Further, it is straightforward to identify these quality levels with those already existing in longestablished analog technologies, such as those in conventional silver-halide photography.

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#### **Biography**

Rodney Shaw received his PhD from Cambridge University. He came to the USA in 1973, and following research appointments at Xerox and Eastman Kodak was Director of the Center for Imaging Science at RIT. He joined H-P Labs in 1994, and his current interests are in digital photography and systems modeling. *rodney shaw@hpl.hp.com*